1. Find the 4x4 homogeneous 3D transformation matrix that translates by \((3, 5, 2)\), rotates by 45 degrees about the Z-axis, and finally, scales by \((3, 1, 2)\).

2. Suppose you wish to apply the transformation in question 1 to the normal vectors in a 3-D model. What is the transformation matrix that needs to be applied to the normal vectors in order to preserve the normals’ perpendicular orientation relative to the surface of the object?

3. (a) What is a *scene graph*? What problem are scene graphs intended to help solve?

   (b) Consider the following 2-D scene of a person standing on a boat rolling in the waves. There are three objects in the scene: the boat, the person, and the person’s hat. The origin of the boat is at \((x, y)\) in the scene. \(\theta\) represents the amount the boat is rolling. \(a\) is the horizontal position of the person on the deck of the boat and \(b\) is the height of the deck (relative to the origin of the boat). \(h\) is the height of the person and \(w\) is the distance from the origin of the person to the midpoint of the person’s body. The hat sits on top of the middle of the person’s head and is tilted with angle \(\varphi\).

   Draw the scene graph for this scene. Write out the transforms that need to be composed in order to obtain the model transform (i.e., transform used to position the 3-D model in the world) for the person’s hat.

4. A triangular face, which is part of a larger 3-D model of a sphere, is defined by vertices at the points \((1, 0, 1)\), \((3, 2, 4)\) and \((5, 3, 1)\)
(a) If the convention is vertices for front-facing triangles are provided anti-clockwise, find the surface normal of the triangle defined by these three vertices.

(b) If the sphere is to be rasterized using surface-normal interpolation, would you expect the normal vector at the vertices to be set to the vector you found in (a)? Why or why not?

5. A triangle with vertices $(0.0, 0.0, 0.0), (1.0, 0.5, −0.5), (−1.0, 3.0, 0.0)$ has the following RGB value mapped to each vertex: $(0.0, 0.0, 0.0), (0.5, 0.0, 0.5)$, and $(0.0, 1.0, 1.0)$, respectively.

If a rasterizer uses interpolation between the vertex colours to calculate the colour within the triangle, What colour is the point $(0.0, 0.875, −0.125)$ inside the triangle?

6. Consider a scene with $V$ vertices, $N$ objects, and $T$ triangles being rendered to a display with $P$ pixels. Assuming a single fragment shader and no anti-aliasing, provide an expression for the upper-bound of how many times the fragment shader will be run using some (or all) of the variables given.

7. A bump/normal-mapped object can be made to appear bumpy when in fact the geometry is smooth. Given a bumpy-looking object, explain what characteristics can be used to determine whether the object has bumpy geometry or is making use of bump-mapping.

8. As you know, double-buffering helps eliminate flicker and tearing.

   (a) What is flicker? What is tearing?

   (b) Give an example of an application where double-buffering may not be desirable.

9. Explain why there are no colours lying outside of the curve formed by the pure colours in the CIE chromaticity diagram.

10. The following are a few questions about colour spaces. One tool for exploring colour spaces available online is [https://colorizer.org/](https://colorizer.org/), which may help with these questions.

    (a) What does the ”H” in HSV stand for? Can you discern how the ”H” values are ordered in the HSV and the other Hxx colour spaces?
(b) What values of CMYK does white correspond to?
(c) What features do the "L", "a" and "b" in the Lab colour space correspond to?

11. Lecture slides 230 and 231 show two possible tone-mappings for a given image.

(a) First, discuss what is different between the two tone maps.
(b) What qualitative differences in the appearance of the displayed image would you expect to see between the first and second tone maps?